

# DYNAMICS OF PRIMORDIAL HYDROGEN RECOMBINATION WITH ALLOWANCE FOR A RECOIL FOR SCATTERING IN THE Ly- $\alpha$ LINE

S.I.Grachev<sup>1</sup>, V.K.Dubrovich<sup>2,3, 4</sup>

<sup>1</sup>*Sobolev Astronomical Institute, Saint Petersburg State University, Saint Petersburg, Russia*

<sup>2</sup>*Special Astrophysical Observatory of RAS, Saint Petersburg branch, Saint Petersburg, Russia*

<sup>3</sup>*The Main Astronomical Observatory of RAS, Saint Petersburg, Russia*

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*Abstract.* It is shown that taking into account a recoil for radiation scattering in the Ly- $\alpha$  line can lead to a noticeable acceleration of primordial hydrogen recombination. Thus for  $\Lambda$ CDM model a decrease of ionization degree exceeds 1% for redshifts  $z$  in a range 800 – 1050 achieving  $\approx 1.3\%$  at  $z = 900$ . Corresponding corrections to the cosmic microwave background power spectra can achieve 1.1% for  $TT$  spectra and 1.7% for  $EE$  ones. Radiative transfer in these calculations was treated in a quasistationary approximation. Numerical solutions are also obtained in diffusion approximation for a nonstationary problem of Ly- $\alpha$  line radiative transfer under partial frequency redistribution with a recoil. An evolution of a local line profile is traced to as well as an evolution of a relative number of uncompensated transitions from 2p state down to 1s one. It is shown that taking into account nonstationarity of Ly- $\alpha$  line radiative transfer can lead to an additional acceleration of primordial hydrogen recombination.

*Key words:* cosmology, early Universe, recombination epoch, primordial hydrogen, cosmic microwave background radiation (CMBR)

## INTRODUCTION

In connexion with the proposed considerable increase of CMBR observations precision the requirements grew as well to predictions accuracy of the theory being used to interpret the observations. In particular this concerns the theory of cosmological recombination. Required accuracy is about 0.1%. So at last time a few papers appear (Dubrovich, Grachev, 2005; Chluba, Sunyaev, 2006; Kholupenko, Ivanchik, 2006; Rubiño-Martin et al., 2006; Chluba et al., 2007; Wong, Scott, 2007; Chluba, Sunyaev, 2007) where recombination history of primordial hydrogen is calculated with a proper allowance for some fine effects which however are essential at the required level of accuracy.

In this paper we study an effect of allowance for a recoil under radiation scattering in Ly- $\alpha$  line on recombination history of primordial hydrogen. Earlier we already carried out calculations of such kind (Grachev, Dubrovich, 1991) for purely baryonic models. An effect of a recoil turned out to be practically negligible. However our calculations fulfilled in the framework of present-day  $\Lambda$ CDM models show that for these models characterized by low baryon density the effect of recoil becomes nearly ten times larger. The reason is in the fact that at low baryon density the relative role of Ly- $\alpha$  photons escape from the process of scattering due to Universe expansion (so called "intrinsic escape") considerably increases as compared with another main mechanism of irreversible recombination, namely, two-photon transitions from

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<sup>4</sup>e-mail: dvk47@mail.ru

the second level. Therefore the role of fine effects in treating of radiation transfer in the Ly- $\alpha$  line grows correspondingly.

The second circumstance to which we pay attention is an influence of nonstationarity of radiation transfer in the main resonance line on recombination history. Up to now a quasi-stationary approximation is in usage. Namely, when computing hydrogen recombination history one makes use of stationary Sobolev approximation for the number of uncompensated transitions from  $2p$  state down to  $1s$  one (so called net radiative bracket (NRB)). However it would be very important to determine characteristic time necessary for NRB to reach a steady-state limit. In this connexion we fulfilled model numerical calculations of time dependent radiative transfer in a resonance line under partial frequency redistribution (PFR) allowing for a recoil in the process of scattering. It turned out that time of reaching a steady-state limit for NRB may well exceed 1% (in  $\Delta z/z$  units) in the range of redshifts near  $z \approx 1000$ .

## RECOMBINATION DYNAMICS ASSUMING QUASI-STATIONARY RADIATIVE TRANSFER

It is known that Sobolev approximation widely used in astrophysics to treat radiative transfer in spectral lines in moving media gives an exact solution of a stationary problem for the case of two-level atoms in an infinite uniform expanding medium assuming complete frequency redistribution (CFR) in a single scattering. Then for a relative number of uncompensated transitions (so called NRB) we have

$$r_{21} \equiv [n_2(A_{21} + B_{21}J_{12}) - n_1B_{12}J_{12}]/n_2A_{21} = \beta(1 - B/S), \quad (1)$$

where

$$\beta = \gamma \left(1 - e^{-1/\gamma}\right) \quad (2)$$

has a sense of photon escape probability out of the process of scattering,

$$\gamma = \frac{8\pi}{\lambda_{12}^3} \frac{H}{A_{21}[(g_2/g_1)n_1 - n_2]} \quad (3)$$

is a dimensionless velocity gradient proportional to velocity gradient  $H$  (which is the Hubble factor in cosmological context). A quantity  $\tau_S = 1/\gamma$  is called Sobolev optical distance. It is an optical depth of an infinite expanding medium along photon path. Above we use standard designations for Einstein coefficients of transition probabilities and for statistical weights of levels. Moreover  $J_{12}$  is the mean intensity of radiation in a line,  $\lambda_{12}$  is a transition wavelength,  $S = (2hc/\lambda_{12}^3)[g_1n_2/(g_2n_1 - g_1n_2)]$  is a line source function,  $B$  is the Planck function at the line center (we suggest that far in the violet wing of the line radiation intensity is planckian and we also ignore frequency dependence of the Planck function within the line). From eqs. (2) and (3) we have equation  $\beta \propto H/\Omega_B$  which clearly demonstrates a growth of escape probability when  $\Omega_B$  decreases.

As it was shown by Chugaj (1980) rejection of CFR i.e. usage of PFR does not affect on a mean number of scatterings and on excitation degree of atoms. Therefore if one neglects induced radiation and suggests that  $\gamma \ll 1$  then the equations given above may be used for PFR (Grachev, 1989; Grachev, Dubrovich, 1991) if only recoil under scattering is neglected. Allowing for recoil leads to the growth of photon escape probability due to additional redshift

of photons. From analytical solution of the problem in diffusion approximation (Grachev, 1989) the following expression is obtained for it suggesting zero boundary condition in a far violet wing:

$$\beta = i(-\infty)\gamma, \quad \gamma \ll 1, \quad (4)$$

where correction factor  $i(-\infty) > 1$ . It depends on parameters

$$\rho = 2\delta x_\gamma, \quad \sigma = x_\gamma/x_\lambda, \quad (5)$$

where  $\delta$  is a recoil parameter and  $x_\gamma$  and  $x_\lambda$  are characteristic dimensionless frequencies:

$$\delta = h\nu_{12}/Mv_{\text{th}}c, \quad x_\gamma = (3a\lambda/2\pi\gamma)^{1/3}, \quad x_\lambda = \sqrt{\lambda/2(1-\lambda)}. \quad (6)$$

Here  $M$  is an atom mass,  $v_{\text{th}} = \sqrt{2kT_e/M}$  is a most probable velocity of atoms thermal motion,  $a$  is the Voigt parameter,  $\lambda$  is an albedo per single scattering (a probability of photon "survival" in a single scattering). Characteristic frequency  $x_\gamma$  was introduced by Chugaj (1980). It separates domain of photons diffusion ( $|x| < x_\gamma$ ) from domain of drift ( $|x| > x_\gamma$ ) due to differential motion (expansion) of the medium (here and so on  $x = (\nu - \nu_{12})/\Delta\nu_D$ ,  $\Delta\nu_D = \nu_{12}v_{\text{th}}/c$ ). Results of numerical calculations of  $i(-\infty)$  can be approximated by equations

$$i(-\infty) \approx 1 + \rho/[1 + (\sigma^2/3)(4 - \sigma^2)/(2 + \sigma^2) + \sigma^4/6], \quad \sigma > 0.38, \quad (7)$$

and

$$i(-\infty) \approx \left[ 3 \int_0^\infty e^{-\rho y - y^3} y^2 dy \right]^{-1}, \quad \sigma < 0.38, \quad (8)$$

with an error less than 0.3%. It should be noted that for  $i(-\infty) - 1$  an error of these equations is less than 8%.

For non-zero boundary condition  $I(+\infty) = B$  in a far violet wing eq. (1) turns then into (Grachev, Dubrovich, 1991)

$$r_{21} = \gamma[i(-\infty) - B/S]. \quad (9)$$

Computations of hydrogen recombination history using eqs. (7) – (9) for  $2p - 1s$  transition were carried out by us earlier in the framework of 60-level model of hydrogen atom for purely baryonic universe models and in particular for values of parameters being used in a pioneer works on cosmological recombination (Zeldovich et al., 1968; Peebles, 1968):  $\Omega_{\text{tot}} = 1$ ,  $\Omega_B = 1$ ,  $H_0 = 98$  km/s/Mpc,  $T_0 = 2.7$  K,  $Y_{\text{He}} = 0$ . Results of these our calculations are shown in Fig. 1 along with the results for  $\Lambda$ CDM model with parameters  $\Omega_{\text{tot}} = 1$ ,  $\Omega_\Lambda = 0.7$ ,  $\Omega_{\text{DM}} = 0.26$ ,  $\Omega_B = 0.04$ ,  $H_0 = 70$  km/s/Mpc,  $T_0 = 2.728$  K,  $Y_{\text{He}} = 0.24$ . Namely, in Fig. 1 there are graphs of relative variation (in percents) of a fractional electron density  $x_e = n_e/n_H$  due to allowance for a recoil under Ly- $\alpha$  photons scattering. (Here  $n_e$  and  $n_H$  are the number densities of electrons and of hydrogen atoms and ions respectively.) As a mechanism of Lyman photons "destruction" per scattering we consider excitation and ionization of atoms from  $2p$  state due to absorption of background black-body radiation. So we have for albedo per a single scattering an expression

$$\lambda = A_{2p,1s} / \left( A_{2p,1s} + R_{2p,c} + \sum_{i=3}^{\infty} R_{2p,i} \right). \quad (10)$$

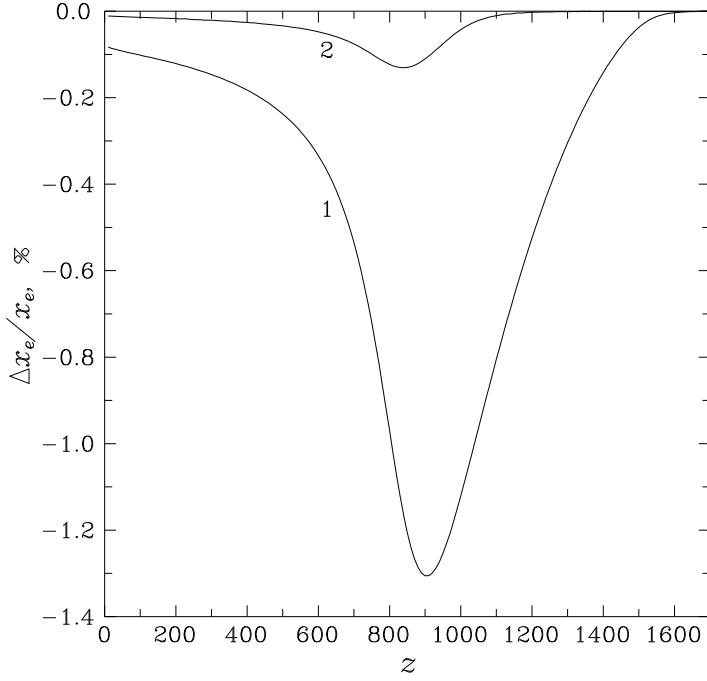


Figure 1: Variation of hydrogen recombination history due to effect of a recoil under scattering in Ly- $\alpha$  line. Parameters values: 1)  $\Omega_{\text{tot}} = 1$ ,  $\Omega_{\Lambda} = 0.7$ ,  $\Omega_{\text{B}} = 0.04$ ,  $\Omega_{\text{DM}} = 0.26$ ,  $H_0 = 70$  km/s/Mpc,  $T_0 = 2.728$  K,  $Y_{\text{He}} = 0.24$ ; 2)  $\Omega_{\text{tot}} = 1$ ,  $\Omega_{\Lambda} = 0$ ,  $\Omega_{\text{B}} = 1$ ,  $\Omega_{\text{DM}} = 0$ ,  $H_0 = 98$  km/s/Mpc,  $T_0 = 2.7$  K,  $Y_{\text{He}} = 0$ .

One can see in Fig. 1 that for a present-day Universe model recoil contribution increases more than ten times and is in the level of 1% near the surface of last scattering.

Variation of primordial hydrogen recombination history has an influence on the power spectra of CMBR angular fluctuations in intensity (temperature) and polarization. Corresponding graphs are shown in Fig. 2. Computations were fulfilled by means of CMBFAST package (Seljak, Zaldarriaga, 1996). It is seen that relative variations can reach 1.1% for  $TT$  spectra and 1.7% for  $EE$  spectra for multipoles  $l < 2500$ .

### NONSTATIONARY RADIATIVE TRANSFER in Ly- $\alpha$ LINE

To clear up how fast NRB reaches a steady-state limit we have solved a model problem of nonstationary radiative transfer in isolated spectral line. It is suggested that at some initial moment of time ( $t = 0$ ) there is no radiation and at this moment primary sources of line radiation with a constant power are switched on. The main integro-differential equation of radiative transfer has a form

$$\frac{\partial I(t, x)}{\partial t} - \gamma \frac{\partial I(t, x)}{\partial x} = -\phi(x)I(t, x) + \lambda \int_{-\infty}^{+\infty} R(x, x')I(t, x')dx' + (1 - \lambda)\phi(x)S_0, \quad (11)$$

where  $I(t, x)$  is a radiation intensity ( $x = (\nu - \nu_{12})/\Delta\nu_{\text{D}}$  is a distance from a line center measured in Doppler widths;  $t$  is a time measured in units of mean time between successive scatterings),  $R(x, x')$  is a frequency redistribution function per scattering,  $\phi(x)$  is an absorption coefficient profile,  $\gamma$  is a dimensionless velocity gradient (see above eq. (3) in which however a contribution of induced radiation should be neglected),  $\lambda$  is an albedo per a single scattering. The following

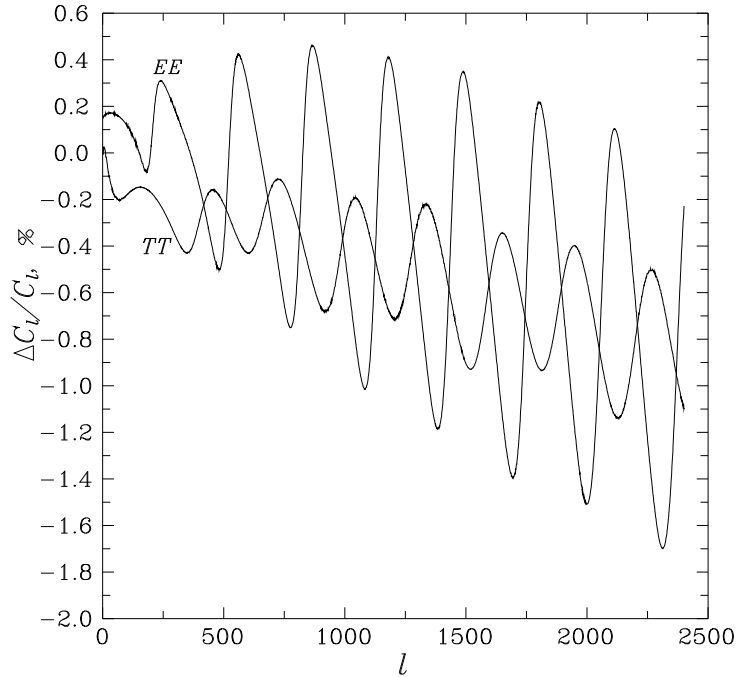


Figure 2: Variation of CMBR power spectra of temperature ( $TT$ ) and polarization ( $EE$ ) fluctuations. Computations are fulfilled with the CMBFAST package (Seljak, Zaldarriaga, 1996). Values of parameters are the same as for the curve 1 in Fig. 1.

normalizations hold

$$\int_{-\infty}^{+\infty} R(x, x') dx = \phi(x'), \quad \int_{-\infty}^{+\infty} \phi(x) dx = 1. \quad (12)$$

Initial and boundary conditions for eq. (11) are  $I(0, x) = 0$ ,  $I(\tau, +\infty) = 0$ .

In the context of cosmological recombination an absorption coefficient profile is the Voigt one with parameter  $a = A_{2p,1s}/4\pi\Delta\nu_D$ . As concerned frequency redistribution function with allowance for a recoil it was derived in the paper by Basko (1981) where diffusion approximation was also obtained for the integral term (in transfer equation) describing an act of a single scattering. Using this approximation one can reduce eq. (11) (with  $S_0 = 1$ ) to the form

$$\frac{\partial I}{\partial \tau} - \frac{\partial I}{\partial y} = - \left( \sigma^2 + \frac{2\rho}{y} \right) \frac{I(\tau, y)}{3y^2} + \frac{1}{3y^2} \left( \rho - \frac{2}{y} \right) \frac{\partial I}{\partial y} + \frac{1}{3y^2} \frac{\partial^2 I}{\partial y^2} + \frac{\sigma^2}{3y^2} + \frac{\sigma^2}{3\varepsilon} [1 - I(\tau, 0)] \delta(y), \quad (13)$$

where new dimensionless frequency and time are  $y = x/x_\gamma$ ,  $\tau = \gamma t/x_\gamma$  and  $\varepsilon = a/\pi x_\gamma$  is a small parameter. Parameters  $\rho$  and  $\sigma$  are defined above by eqs. (5) and (6). When obtaining eq. (13) the profile of absorption coefficient was taken approximately in the form (Grachev, 1989)

$$\phi(x) = \delta(x) + a/\pi x^2, \quad (14)$$

where the first term ( $\delta$ -function) describes Doppler core and the second one corresponds to the Lorentz wings of the Voigt profile.

In a steady-state case (when  $\partial I/\partial \tau = 0$ ) analytical solution of eq. (13) was constructed by Grachev (1989) and then has been used (Grachev, Dubrovich, 1991) to compute recombination history of primordial hydrogen (see also above in the preceeding section). In the present paper we obtain numerical solutions of nonstationary eq. (13) for the values of parameters typical

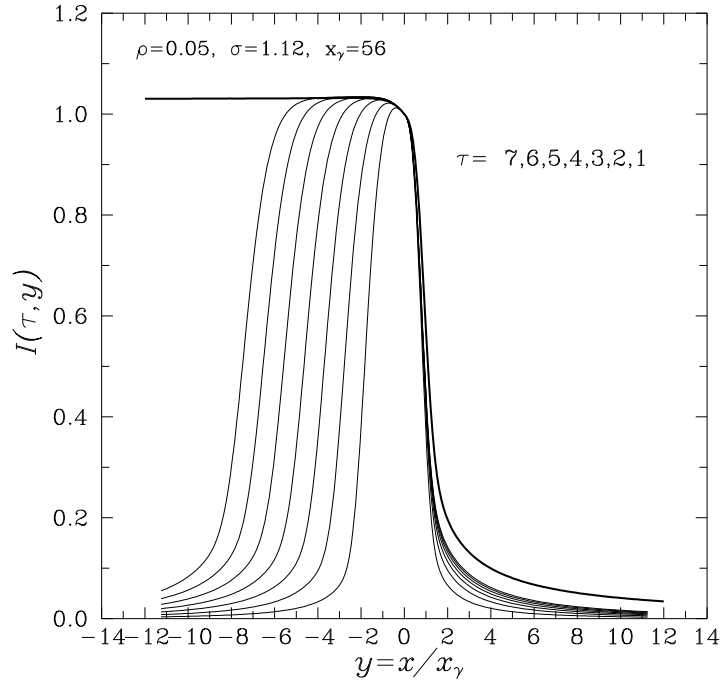


Figure 3: Evolution of a local Ly- $\alpha$  line profile in the case of non-conservative scattering. An upper profile corresponds to  $\tau = \infty$ . Values of model parameters are the same as for the curve 1 in Fig. 1.

for cosmological recombination. (As a matter of fact the function  $f(\tau, y) = \int_y^\infty I(\tau, y') dy'$  was found at first from equation which is easily obtained by integrating eq. (13).) We make use of numerical method of solving nonstationary problems of radiative transfer theory proposed by Grachev (2001). Results are in Fig. 3. The values of  $\Lambda$ CDM model parameters are the same as for the curve 1 in Fig. 1. As for parameters depending on redshift  $z$  their values were calculated by means of our code which computes hydrogen recombination history (Grachev, Dubrovich, 1991). The point  $z = 1180$  near the surface of last scattering was taken. At this point we have  $x_e = 0.300$ ,  $1 - \lambda = 2.00 \cdot 10^{-4}$ ,  $a = 8.30 \cdot 10^{-4}$ ,  $\gamma = 2.26 \cdot 10^{-9}$ ,  $x_\gamma = 56.0$ ,  $\varepsilon = 5 \cdot 10^{-6}$ ,  $\rho = 0.0500$ ,  $\sigma = 1.12$ ,  $i(-\infty) - 1 = 0.0296$ . For comparison in Fig. 4 results of similar calculations are shown but for  $\sigma = 0$  which corresponds to conservative scattering ( $\lambda = 1$ ) (then  $i(-\infty) - 1 = 0.0455$ ). (Since in this case eq. (13) becomes homogeneous we normalize its solution to unity at  $\tau = \infty$  and  $x/x_\gamma = -\infty$ .) One can see that in both cases the profiles approach asymptotically to a steady state limit which corresponds to the upper curves on Figs. 3 and 4.

For NRB one can obtain directly from eq. (11) the following expression

$$r_{21} = \beta = \frac{\gamma}{1 - \lambda + \lambda J(\tau)} \frac{\partial}{\partial \tau} \int_{-\infty}^{+\infty} I(\tau, y) dy, \quad (15)$$

where

$$J(\tau) = \int_{-\infty}^{+\infty} \phi(x) I(\tau, x) dx. \quad (16)$$

According to eq. (14) it follows from here that  $J(\tau) = I(\tau, 0) + O(\varepsilon)$ . Fig. 5 displays that approaching of  $r_{21}$  to the stationary limit (dashed horizontal lines) is going on quite slowly especially for non-conservative scattering ( $\sigma = 1.12$ ). The point is that for non-conservative

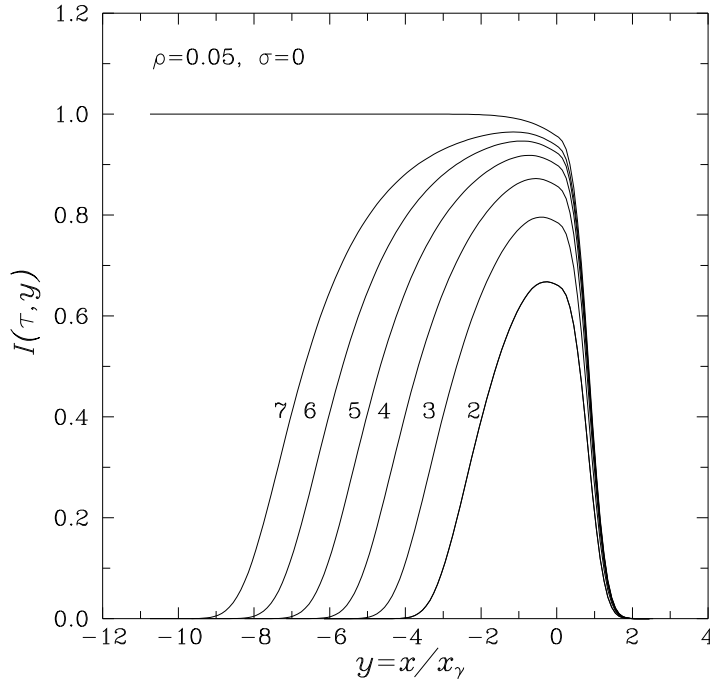


Figure 4: The same as in the preceeding Fig. but for conservative scattering. Numbers near the curves are the moments of time  $\tau$ .

scattering the line profile has extended quasi-lorentzian wings (one can show that  $I(\tau, y) \sim \sigma^2 \tau [3y(y + \tau)]^{-1}$  for  $|y| \gg 1$ ,  $|y + \tau| \gg 1$ ) which evolve very slowly towards the steady state limit ( $I(\infty, y) \sim \sigma^2/3y$ ) for  $y \gg 1$  (see Fig. 3). According to Fig. 5 the time to reach steady-state limit can be as large as  $\tau \approx 10$  and even more. An interval of redshifts  $\Delta z$  corresponding to this time can be estimated from equation  $dz/dt = -(1 + z)H(z)$  from which it follows that

$$\frac{\Delta z}{1 + z} = \frac{v_{\text{th}}(z)}{c} x_\gamma(z) \Delta \tau. \quad (17)$$

From here one can obtain that  $\Delta z/z = 1.4 \cdot 10^{-3} \Delta \tau$  for given above values of parameters at  $z = 1180$ . Thus non-stationarity of radiative transfer in Ly- $\alpha$  line can lead to an appreciable acceleration of irreversible recombination of primordial hydrogen.

From physical point of view an increase of NRB due to non-steady state radiative transfer takes place because the line does not reach a saturation in some frequency regions after primary sources switch on, i.e. there are not enough resonance photons in these regions to compensate in a maximum degree radiative transitions down by those of up.

## CONCLUSION

In this paper we study an effect of a more accurate treating of radiative transfer in Ly- $\alpha$  line on a calculated recombination history of primordial hydrogen. An urgent necessity of such kind investigation is connected with increased role of Lyman photons escape from a scattering process in the framework of present day  $\Lambda$ CDM models characterized by low baryon density. We show that allowance for a recoil under scattering in Ly- $\alpha$  line can lead to an appreciable acceleration of primordial hydrogen acceleration. As a main one we consider a model with the following values of parameters  $\Omega_{\text{tot}} = 1$ ,  $\Omega_\Lambda = 0.7$ ,  $\Omega_{\text{DM}} = 0.26$ ,  $\Omega_B = 0.04$ ,  $H_0 = 70$

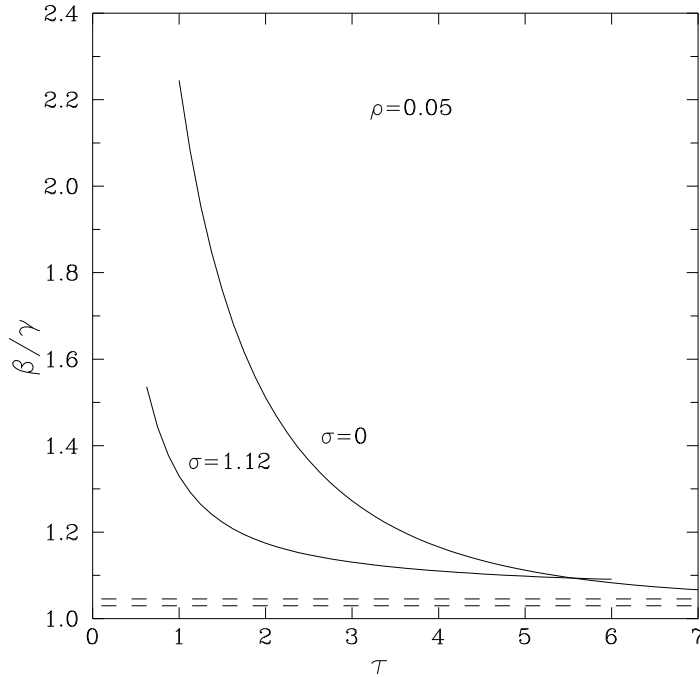


Figure 5: Ly- $\alpha$  photons escape probability (or NRB) normalized to Sobolev value. Dashed horizontal lines correspond to the steady-state limit allowing for a recoil (upper line:  $\sigma = 0$ , lower line:  $\sigma = 1.12$ ).

km/s/Mpc,  $T_0 = 2.728$  K,  $Y_{\text{He}} = 0.24$ . For this model a decrease of hydrogen ionization degree exceeds 1% in the range of redshifts  $z = 800 - 1050$  reaching  $\approx 1.3\%$  at  $z = 900$ . Corresponding variations in computed power spectra of CMBR angular fluctuations reach 1.1% for  $TT$  spectra and 1.7% for  $EE$  spectra for multipoles  $l < 2500$ . It should be stressed that radiative transfer in these calculations is treated (as well as in other works on cosmological recombination of hydrogen) in a quasi-stationary approximation. In order to study an applicability of this approximation we have obtained numerical solutions of nonstationary problem of radiative transfer in Ly- $\alpha$  line assuming partial (true) frequency redistribution with a proper allowance for a recoil. An evolution of a local line profile is traced through as well as an evolution of number of uncompensated transitions from  $2p$  state down to  $1s$  one. It is shown that taking into account non-stationarity of radiative transfer in Ly- $\alpha$  line can lead to an appreciable additional acceleration of primordial hydrogen recombination.

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